MISIDENTIFICATION OF DYNAMICAL SYSTEMS CAUSED BY NUMERICAL INTEGRATORS

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In this presentation, I will introduce the ODEnet—a framework that incorporates traditional numerical methods into loss function design. One key application is to extract the underlying dynamical system from available observation snapshots. For an autonomous differential system given by $dx/dt = f(x)$, the ODEnet approximates the unknown function $f(x)$ with a trainable neural network while reducing the discrepancy between numerical predictions and the observations at specific sampling times. My focus will be on the example of learning linear dynamical systems, whether they are conservative or dissipative.

The first part of the talk explores structural challenges, such as rotation and drift in the learned dynamical systems, using one-step and multi-step methods. Although the loss function might be minimized ideally, the resultant models may not fully capture the true behavior of the system. I will share our findings and propose criteria for selecting numerical integrators that preserve the inherent structure of the unknown dynamics.

In the second section, I will examine the effects of noise on the learning outcomes, demonstrating how different noise levels can distort the results compared to those derived from noise-free data.

The third section will discuss the application of Richardson extrapolation to improve the accuracy of the learned dynamics, moving them closer to the actual underlying system. I will also present sufficient conditions that ensure structural preservation and enhanced accuracy.

Time permitting, I will conclude by presenting preliminary results on learning nonlinear dynamical systems, which extend our comprehensive understanding of the linear case, and share some numerical outcomes from these studies.

Key Words: dynamical learning problem, ODEnet

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